XXV OLIMPIADA MATEMÁTICA DE CENTROAMÉRICA Y EL CARIBE
EL SALVADOR 2023

English, Day 1

La Herradura, La Paz

Tuesday, July 25th, 2023

1. A *coloring* of the set of integers greater than or equal to 1, must be done according to the following rule: Each number is colored blue or red, so that the sum of any two numbers (not necessarily different) of the same color is blue. Determine all the possible *colorings* of the set of integers greater than or equal to 1 that follow this rule.

2. Octavio writes an integer $n \geq 1$ on a blackboard and then starts a process in which, at each step he erases the integer k written on the blackboard and replaces it with one of the following numbers:

$$3k - 1, \quad 2k + 1, \quad \frac{k}{2},$$

provided that the result is an integer.

Show that for any integer $n \geq 1$, Octavio can write on the blackboard the number 3^{2023} after a finite number of steps.

3. Let a, b and c be positive real numbers such that $ab + bc + ca = 1$. Show that

$$\frac{a^3}{a^2 + 3b^2 + 3ab + 2bc} + \frac{b^3}{b^2 + 3c^2 + 3bc + 2ca} + \frac{c^3}{c^2 + 3a^2 + 3ca + 2ab} > \frac{1}{6(a^2 + b^2 + c^2)^2}.$$

Language: English

Time: 4 hours 30 minutes
Each problem is worth 7 points