XXV OLIMPIADA MATEMÁTICA DE CENTROAMÉRICA Y EL CARIBE
EL SALVADOR 2023

English, Day 2

La Herradura, La Paz

Wednesday, July 26^{th}, 2023

4. A four-digit number $n = \overline{abcd}$, where a, b, c and d are digits, with $a \neq 0$, is called *guanaco* if the product $\overline{ab} \times \overline{cd}$ is a positive divisor of n. Find all the *guanaco* numbers that exist.

5. Let ABC be an acute-angled triangle with $AB < AC$ and let Γ be the circle passing through A, B and C. Let D be the point diametrically opposite to A in Γ and let ℓ be the tangent line at D to Γ. Let P, Q and R be the points of intersection of BC with line ℓ, of AP with Γ such that $Q \neq A$, and of QD with the altitude of triangle ABC through A, respectively. Point S is defined as the intersection of AB with the line ℓ and point T is the intersection of AC with the line ℓ. Prove that points S and T belong to the circle passing through A, Q and R.

6. In a pond there are $n \geq 3$ stones arranged on a circle.

A princess wants to label the stones with the numbers $1, 2, \ldots, n$ in some order and then place some frogs on the stones. Once all the frogs are placed, they start jumping clockwise according to the following rule: When a frog reaches the stone labeled with the number k, it waits k minutes and then jumps to the adjacent stone.

If at no time two frogs can occupy the same stone, what is the largest number of frogs for which the princess can label the stones before placing the frogs? For this maximum, show how the princess can label the stones and how she can place the frogs.

Note. A stone is considered occupied by two frogs at the same time only if there are two frogs that are simultaneously on this stone for at least one minute.

Language: English

Time: 4 hours 30 minutes
Each problem is worth 7 points